Heat and Mass Transfer: Fundamentals & Applications
Fourth Edition
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Chapter 7

EXTERNAL FORCED CONVECTION

Objectives

- Distinguish between internal and external flow
- Develop an intuitive understanding of friction drag and pressure drag, and evaluate the average drag and convection coefficients in external flow
- Evaluate the drag and heat transfer associated with flow over a flat plate for both laminar and turbulent flow
- Calculate the drag force exerted on cylinders during cross flow, and the average heat transfer coefficient
- Determine the pressure drop and the average heat transfer coefficient associated with flow across a tube bank for both in-line and staggered configurations

DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

- Fluid flow over solid bodies frequently occurs in practice such as the *drag force* acting on the automobiles, power lines, trees, and underwater pipelines; the *lift* developed by airplane wings; *upward draft* of rain, snow, hail, and dust particles in high winds; and the *cooling* of metal or plastic sheets, steam and hot water pipes, and extruded wires.
- **Free-stream velocity:** The velocity of the fluid relative to an immersed solid body sufficiently far from the body.
- It is usually taken to be equal to the **upstream velocity** V (**approach velocity**) which is the velocity of the approaching fluid far ahead of the body.
- The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface.

FIGURE 7–1
Flow over bodies is commonly encountered in practice.

Friction and Pressure Drag

- **Drag:** The force a flowing fluid exerts on a body in the flow direction.
- The components of the pressure and wall shear forces in the *normal* direction to flow tend to move the body in that direction, and their sum is called **lift**.
- Both the skin friction (wall shear) and pressure contribute to the drag and the lift.

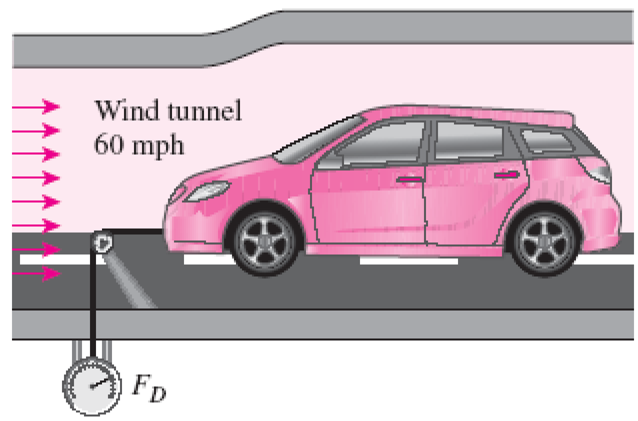


FIGURE 7–2
Schematic for measuring the drag force acting on a car in a wind tunnel.

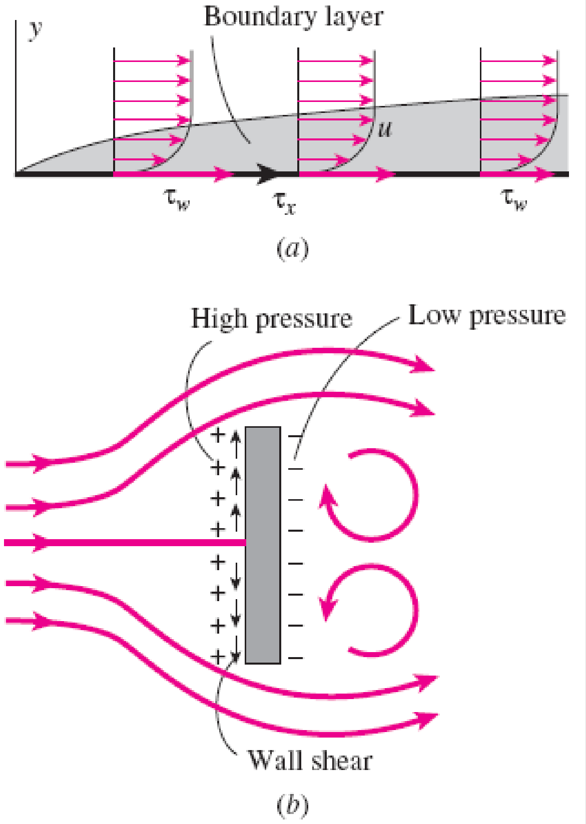


FIGURE 7–3
(a) Drag force acting on a flat plate parallel to the flow depends on wall shear only. *(b)* Drag force acting on a flat plate normal to the flow depends on the pressure only and is independent of the wall shear, which acts normal to the free-stream flow.

The drag force F_D depends on the density of the fluid, the upstream velocity V, and the size, shape, and orientation of the body, among other things.

The drag characteristics of a body is represented by the dimensionless **drag coefficient C_D** defined as

Drag coefficient:
$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

The part of drag that is due directly to wall shear stress τ_w is called the **skin friction drag** (or just *friction drag*) since it is caused by frictional effects, and the part that is due directly to pressure P is called the **pressure drag.**

$$C_D = C_{D,\,\text{friction}} + C_{D,\,\text{pressure}}$$

Flat plate:
$$C_D = C_{D,\,\text{friction}} = C_f$$

$$C_{D,\text{pressure}} = 0$$
$$C_D = C_{D,\text{friction}} = C_f$$

$$F_{D,\text{pressure}} = 0$$
$$F_D = F_{D,\text{friction}} = F_f = C_f A \frac{\rho V^2}{2}$$

FIGURE 7–4
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

- At low Reynolds numbers, most drag is due to friction drag.
- The friction drag is proportional to the surface area.
- The pressure drag is proportional to the frontal area and to the *difference* between the pressures acting on the front and back of the immersed body.
- The pressure drag is usually dominant for blunt bodies and negligible for streamlined bodies.
- When a fluid separates from a body, it forms a separated region between the body and the fluid stream.
- **Separated region:** The low-pressure region behind the body here recirculating and backflows occur.
- The larger the separated region, the larger the pressure drag.

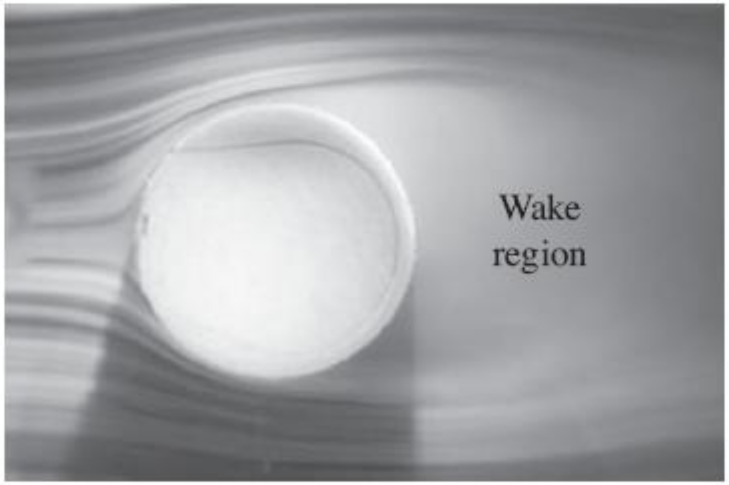


FIGURE 7–5
Separation during flow over a tennis ball and the wake region.

Wake: The region of flow trailing the body where the effects of the body on velocity are felt.

Viscous and rotational effects are the most significant in the boundary layer, the separated region, and the wake.

Heat Transfer

Local and average Nusselt numbers:

$$\mathrm{Nu}_x = f_1(x^*, \mathrm{Re}_x, \mathrm{Pr}) \qquad \text{and} \qquad \mathrm{Nu} = f_2(\mathrm{Re}_L, \mathrm{Pr})$$

Average Nusselt number:

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$$

Film temperature:

$$T_f = \frac{T_s + T_\infty}{2}$$

Average friction coefficient:

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$$

Average heat transfer coefficient:

$$h = \frac{1}{L}\int_0^L h_x\,dx$$

The heat transfer rate:

$$\dot{Q} = hA_s(T_s - T_\infty)$$

PARALLEL FLOW OVER FLAT PLATES

The transition from laminar to turbulent flow depends on the *surface geometry*, *surface roughness*, *upstream velocity*, *surface temperature*, and the *type of fluid*, among other things, and is best characterized by the Reynolds number.

The Reynolds number at a distance *x* from the leading edge of a flat plate is expressed as

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$

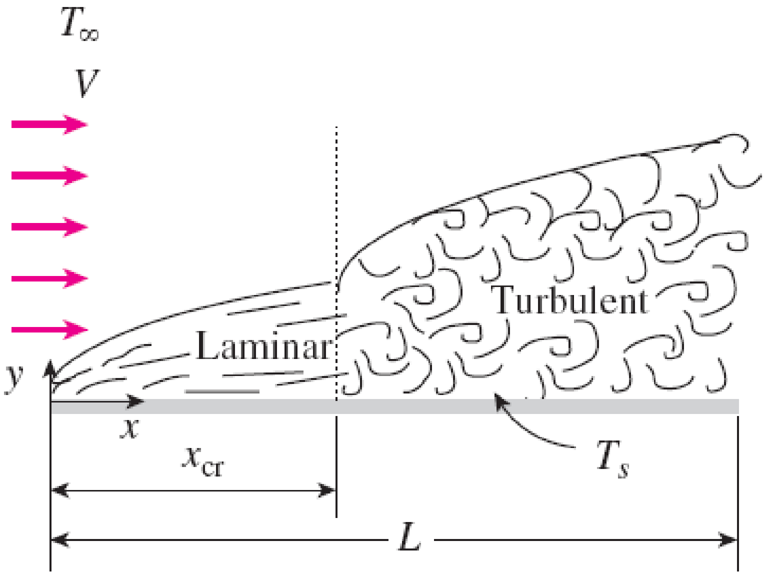


FIGURE 7–6
Laminar and turbulent regions of the boundary layer during flow over a flat plate.

A generally accepted value for the Critical Reynold number

$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from 10^5 to 3×10^6, depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

Friction Coefficient

$$\mathrm{Re}_x = Vx/\nu$$

Laminar: $\delta = \dfrac{4.91x}{\mathrm{Re}_x^{1/2}}$ and $C_{f,x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}}$, $\mathrm{Re}_x < 5 \times 10^5$

Turbulent: $\delta = \dfrac{0.38x}{\mathrm{Re}_x^{1/5}}$ and $C_{f,x} = \dfrac{0.059}{\mathrm{Re}_x^{1/5}}$, $5 \times 10^5 \leq \mathrm{Re}_x \leq 10^7$

Laminar: $C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}}$ $\quad \mathrm{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}}$ $\quad 5 \times 10^5 \leq \mathrm{Re}_L \leq 10^7$

Combined laminar + turbulent flow:

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x\ \text{laminar}}\, dx + \int_{x_{cr}}^{L} C_{f,x,\ \text{turbulent}}\, dx\right)$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \leq \mathrm{Re}_L \leq 10^7$$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log \dfrac{\varepsilon}{L}\right)^{-2.5}$

$$\begin{aligned} C_f &= \frac{1}{L}\int_0^L C_{f,x}\, dx \\ &= \frac{1}{L}\int_0^L \frac{0.664}{\mathrm{Re}_x^{1/2}}\, dx \\ &= \frac{0.664}{L}\int_0^L \left(\frac{Vx}{\nu}\right)^{-1/2} dx \\ &= \frac{0.664}{L}\left(\frac{V}{\nu}\right)^{-1/2} \frac{x^{1/2}}{\frac{1}{2}}\Bigg|_0^L \\ &= \frac{2 \times 0.664}{L}\left(\frac{V}{\nu L}\right)^{-1/2} \\ &= \frac{1.328}{\mathrm{Re}_L^{1/2}} \end{aligned}$$

FIGURE 7–7
The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface. The values shown here are for a laminar flat plate boundary layer.

Relative roughness, ε/L	Friction coefficient C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

*Smooth surface for Re = 10^7. Others calculated from Eq. 7–18 for fully rough flow.

FIGURE 7–8

For turbulent flow, surface roughness may cause the friction coefficient to increase severalfold.

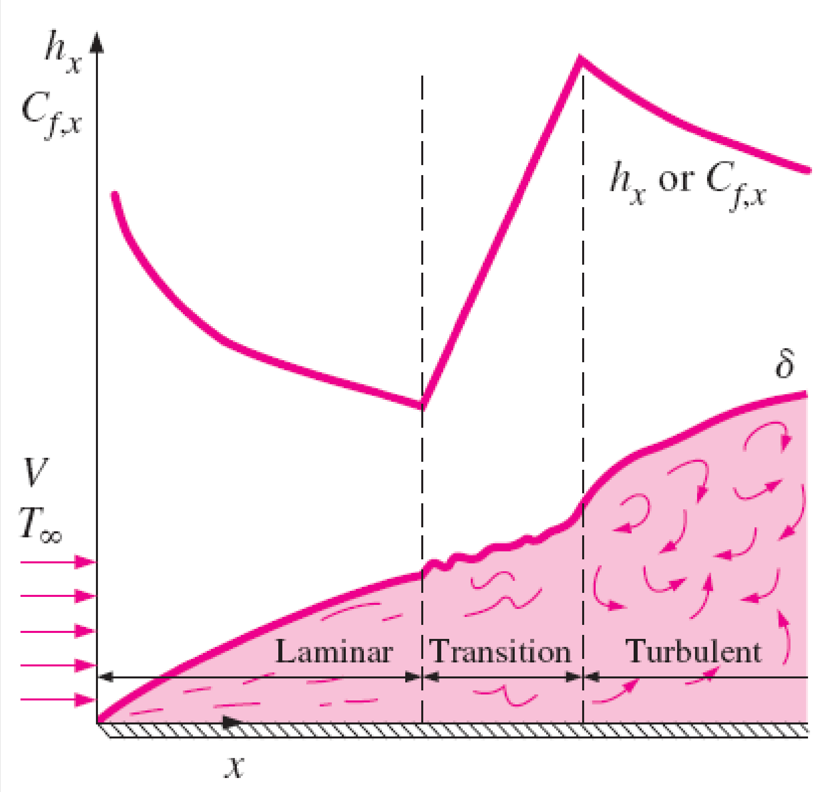
Rough surface, turbulent: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

Heat Transfer Coefficient

The local Nusselt number at a location *x* for laminar flow over a flat plate may be obtained by solving the differential energy equation to be

Laminar: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

Turbulent: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{c} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$



These relations are for *isothermal* and *smooth* surfaces

The local friction and heat transfer coefficients are higher in turbulent flow than they are in laminar flow.

Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction.

The variation of the local friction and heat transfer coefficients for flow over a flat plate.

Nusselt numbers for average heat transfer coefficients

Laminar: $\quad \text{Nu} = \dfrac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} \qquad \text{Re}_L < 5 \times 10^5$

Turbulent: $\quad \text{Nu} = \dfrac{hL}{k} = 0.037\ \text{Re}_L^{0.8}\ \text{Pr}^{1/3} \qquad 0.6 \le \text{Pr} \le 60, \quad 5 \times 10^5 \le \text{Re}_L \le 10^7$

Laminar + turbulent

$$\text{Nu} = \frac{hL}{k} = (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \qquad 0.6 \le \text{Pr} \le 60, \quad 5 \times 10^5 \le \text{Re}_L \le 10^7$$

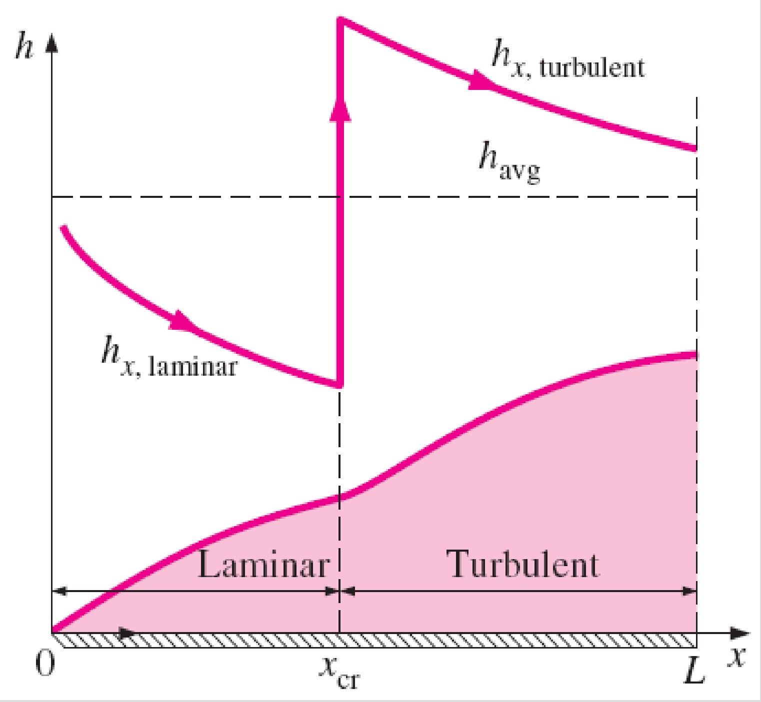
$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\text{ laminar}}\, dx + \int_{x_{cr}}^{L} h_{x,\text{ turbulent}}\, dx\right)$$

For liquid metals

$$\text{Nu}_x = 0.565(\text{Re}_x\ \text{Pr})^{1/2} \qquad \text{Pr} < 0.05$$

For all liquids, all Prandtl numbers

$$\text{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\ \text{Pr}^{1/3}\ \text{Re}_x^{1/2}}{[1 + (0.0468/\text{Pr})^{2/3}]^{1/4}}$$



Graphical representation of the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

It is desirable to have a single correlation that applies to *all fluids*, including liquid metals. By curve-fitting existing data, Churchill and Ozoe (1973) proposed the following relation which is applicable for *all Prandtl numbers* and is claimed to be accurate to $\pm 1\%$,

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \qquad \mathrm{Re}_x\mathrm{Pr} \geq 100 \tag{7–26}$$

$\mathrm{Pe}_x = \mathrm{Re}_x\mathrm{Pr}$ is the dimensionless **Peclet number**

FIGURE 7–11
Jean Claude Eugene Peclet (1793–1857), a French physicist, was born in Besancon, France. He was one of the first scholars of the Ecole Normale at Paris. His publications were famous for their clarity of style, sharp minded views and well performed experiments. The dimensionless **Peclet number** is named after him.

Flat Plate with Unheated Starting Length

Local Nusselt numbers

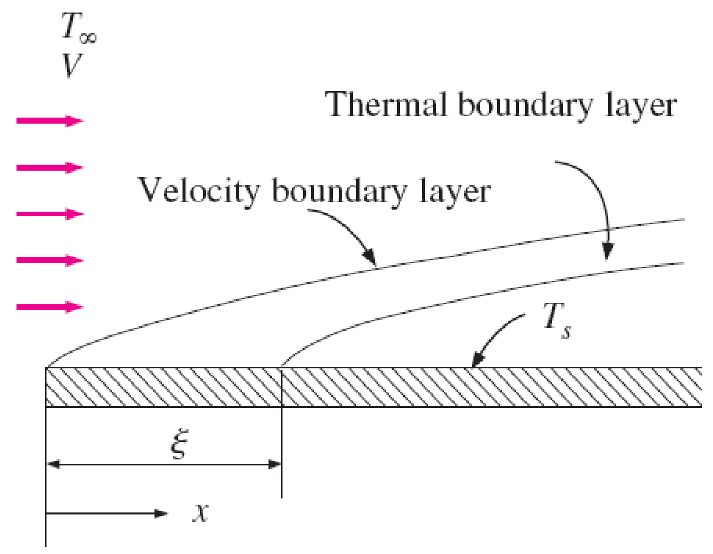
Laminar: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

Turbulent: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

Average heat transfer coefficients

Laminar: $$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent: $$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L}$$



Flow over a flat plate with an unheated starting length.

Uniform Heat Flux

For a flat plate subjected to *uniform heat flux*

$$\textit{Laminar:} \quad \mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6, \qquad \mathrm{Re}_x < 5 \times 10^5$$

$$\textit{Turbulent:} \quad \mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60, \qquad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case.

When heat flux is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance *x* are determined from

$$\dot{Q} = \dot{q}_s A_s$$

$$\dot{q}_s = h_x[T_s(x) - T_\infty] \qquad \rightarrow \qquad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$

EXAMPLE 7–1 Flow of Hot Oil over a Flat Plate

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. 7–12). Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

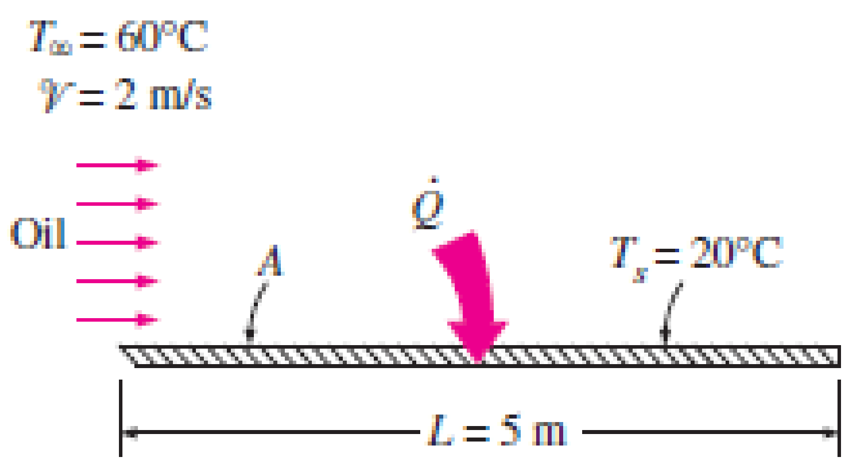


FIGURE 7–12

Schematic for Example 7-1.

SOLUTION Engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$.

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°C$ are (Table A–14).

$$\rho = 876 \text{ kg/m}^3 \qquad \Pr = 2870$$
$$k = 0.144 \text{ W/m}\cdot°\text{C} \qquad \nu = 242 \times 10^{-6} \text{ m}^2\text{/s}$$

Analysis Noting that $L = 5$ m, the Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2\text{/s}} = 4.13 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

$$C_f = 1.328 \, \mathrm{Re}_L^{-0.5} = 1.328 \times (4.13 \times 10^3)^{-0.5} = 0.0207$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the plate per unit width becomes

$$F_D = C_f A_s \frac{\rho \mathcal{V}^2}{2} = 0.0207 \times (5 \times 1 \text{ m}^2) \frac{(876 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)$$
$$= 181 \text{ N}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

This force per unit width corresponds to the weight of a mass of about 18 kg. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 18-kg mass from dropping.

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

Then,

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.144\ \mathrm{W/m \cdot {}^\circ C}}{5\ \mathrm{m}}(1918) = 55.2\ \mathrm{W/m^2 \cdot {}^\circ C}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2\ \mathrm{W/m^2 \cdot {}^\circ C})(5 \times 1\ \mathrm{m^2})(60 - 20)^\circ\mathrm{C} = 11{,}040\ \mathrm{W}$$

EXAMPLE 7–2 Cooling of a Hot Block by Forced Air at High Elevation

The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m × 6 m flat plate whose temperature is 140°C (Fig. 7-13). Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 6-m-long side and (*b*) the 1.5-m side.

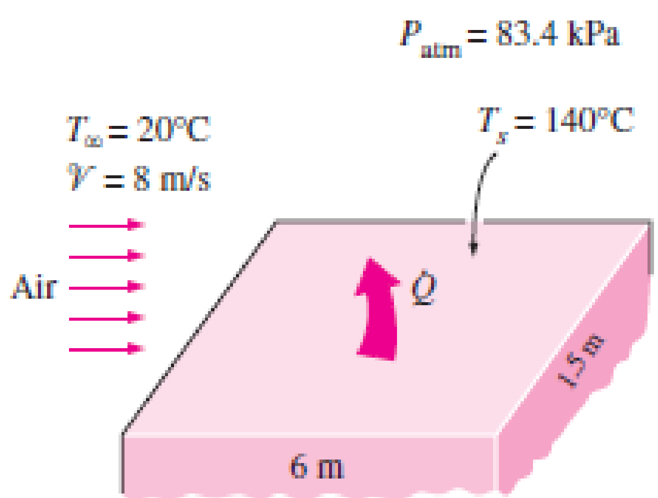


FIGURE 7–13
Schematic for Example 7-2.

SOLUTION The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas.

Properties The properties k, μ, C_p, and Pr of ideal gases are independent of pressure, while the properties ν and α are inversely proportional to density and thus pressure. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (140 + 20)/2 = 80°C$ and 1 atm pressure are (Table A–15)

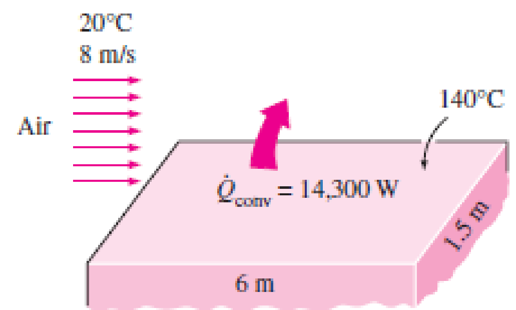
$$k = 0.02953 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu_{@\,1\text{ atm}} = 2.097 \times 10^{-5} \text{ m}^2\text{/s}$$

The atmospheric pressure in Denver is $P = (83.4 \text{ kPa})/(101.325 \text{ kPa/atm}) = 0.823$ atm. Then the kinematic viscosity of air in Denver becomes

$$\nu = \nu_{@\,1\text{ atm}}/P = (2.097 \times 10^{-5} \text{ m}^2\text{/s})/0.823 = 2.548 \times 10^{-5} \text{ m}^2\text{/s}$$

Analysis (*a*) When air flow is parallel to the long side, we have $L = 6$ m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8 \text{ m/s})(6 \text{ m})}{2.548 \times 10^{-5} \text{ m}^2\text{/s}} = 1.884 \times 10^6$$



(*a*) Flow along the long side

FIGURE 7–14
The direction of fluid flow can have a significant effect on convection heat transfer.

which is greater than the critical Reynolds number. Thus, we have combined laminar and turbulent flow, and the average Nusselt number for the entire plate is determined to be

$$\begin{aligned} \text{Nu} = \frac{hL}{k} &= (0.037 \text{ Re}_L^{0.8} - 871)\text{Pr}^{1/3} \\ &= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3} \\ &= 2687 \end{aligned}$$

Then

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953 \text{ W/m}\cdot{}^\circ\text{C}}{6 \text{ m}}(2687) = 13.2 \text{ W/m}^2\cdot{}^\circ\text{C}$$

$$A_s = wL = (1.5 \text{ m})(6 \text{ m}) = 9 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2 \text{ W/m}^2\cdot{}^\circ\text{C})(9 \text{ m}^2)(140 - 20)^\circ\text{C} = 1.43 \times 10^4 \text{ W}$$

Note that if we disregarded the laminar region and assumed turbulent flow over the entire plate, we would get Nu = 3466 from Eq. 7–22, which is 29 percent higher than the value calculated above.

(*b*) When air flow is along the short side, we have $L = 1.5$ m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8 \text{ m/s})(1.5 \text{ m})}{2.548 \times 10^{-5} \text{ m}^2/\text{s}} = 4.71 \times 10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate, and the average Nusselt number is

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (4.71 \times 10^5)^{0.5} \times 0.7154^{1/3} = 408$$

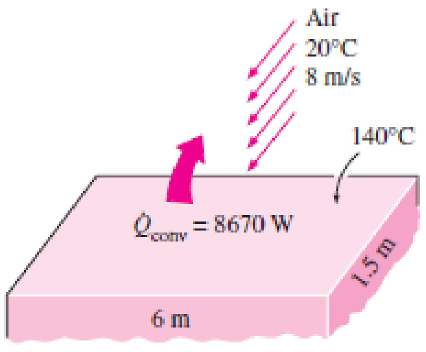
Then

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953 \text{ W/m}\cdot{}^\circ\text{C}}{1.5 \text{ m}}(408) = 8.03 \text{ W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03 \text{ W/m}^2\cdot{}^\circ\text{C})(9 \text{ m}^2)(140 - 20)^\circ\text{C} = 8670 \text{ W}$$

which is considerably less than the heat transfer rate determined in case (*a*).



(*b*) Flow along the short side

FIGURE 7–14
The direction of fluid flow can have a significant effect on convection heat transfer.

EXAMPLE 7–3 Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 4 ft wide and 0.04 in. thick at a velocity of 30 ft/min. The temperature of the plastic sheet is 200°F when it is exposed to the surrounding air, and a 2-ft-long section of the plastic sheet is subjected to air flow at 80°F at a velocity of 10 ft/s on both sides along its surfaces normal to the direction of motion of the sheet, as shown in Figure 7–15. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be ρ = 75 lbm/ft^3, C_p = 0.4 Btu/lbm · °F, and ε = 0.9.

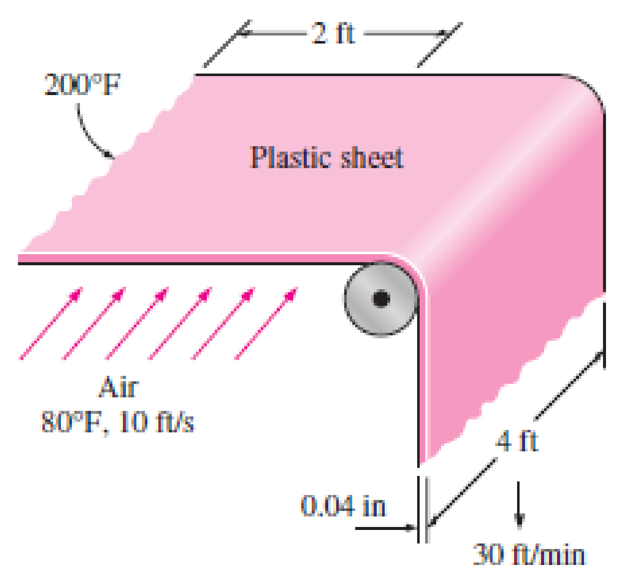


FIGURE 7–15
Schematic for Example 7–3.

SOLUTION Plastic sheets are cooled as they leave the forming section of a plastics plant. The rate of heat loss from the plastic sheet by convection and radiation and the exit temperature of the plastic sheet are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. **3** Air is an ideal gas. **4** The local atmospheric pressure is 1 atm. **5** The surrounding surfaces are at the temperature of the room air.

Properties The properties of the plastic sheet are given in the problem statement. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (200 + 80)/2 = 140°F$ and 1 atm pressure are (Table A–15E)

$$k = 0.01623 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F} \qquad \text{Pr} = 0.7202$$

$$\nu = 0.7344 \text{ ft}^2/\text{h} = 0.204 \times 10^{-3} \text{ ft}^2/\text{s}$$

Analysis (*a*) We expect the temperature of the plastic sheet to drop somewhat as it flows through the 2-ft-long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at 200°F to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet.

Noting that $L = 4$ ft, the Reynolds number at the end of the air flow across the plastic sheet is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(10 \text{ ft/s})(4 \text{ ft})}{0.204 \times 10^{-3} \text{ ft}^2/\text{s}} = 1.961 \times 10^5$$

which is less than the critical Reynolds number. Thus, we have *laminar flow* over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (1.961 \times 10^5)^{0.5} \times (0.7202)^{1/3} = 263.6$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.01623 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}}{4 \text{ ft}}(263.6) = 1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

$$A_s = (2 \text{ ft})(4 \text{ ft})(2 \text{ sides}) = 16 \text{ ft}^2$$

and

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty) \\ &= (1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(16 \text{ ft}^2)(200 - 80){}^\circ\text{F} \\ &= 2054 \text{ Btu/h}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \\ &= (0.9)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)(16 \text{ ft}^2)[(660 \text{ R})^4 - (540 \text{ R})^4] \\ &= 2584 \text{ Btu/h}\end{aligned}$$

Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 2054 + 2584 = \mathbf{4638 \text{ Btu/h}}$$

(*b*) To find the temperature of the plastic sheet at the end of the cooling section, we need to know the mass of the plastic rolling out per unit time (or the mass flow rate), which is determined from

$$\dot{m} = \rho A_c \mathcal{V}_{\text{plastic}} = (75 \text{ lbm/ft}^3)\left(\frac{4 \times 0.04}{12} \text{ ft}^3\right)\left(\frac{30}{60} \text{ ft/s}\right) = 0.5 \text{ lbm/s}$$

Then, an energy balance on the cooled section of the plastic sheet yields

$$\dot{Q} = \dot{m}C_p(T_2 - T_1) \quad \rightarrow \quad T_2 = T_1 + \frac{\dot{Q}}{\dot{m}C_p}$$

Noting that $\dot{Q}$ is a negative quantity (heat loss) for the plastic sheet and substituting, the temperature of the plastic sheet as it leaves the cooling section is determined to be

$$T_2 = 200{}^\circ\text{F} + \frac{-4638 \text{ Btu/h}}{(0.5 \text{ lbm/s})(0.4 \text{ Btu/lbm} \cdot {}^\circ\text{F})}\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = \mathbf{193.6{}^\circ F}$$

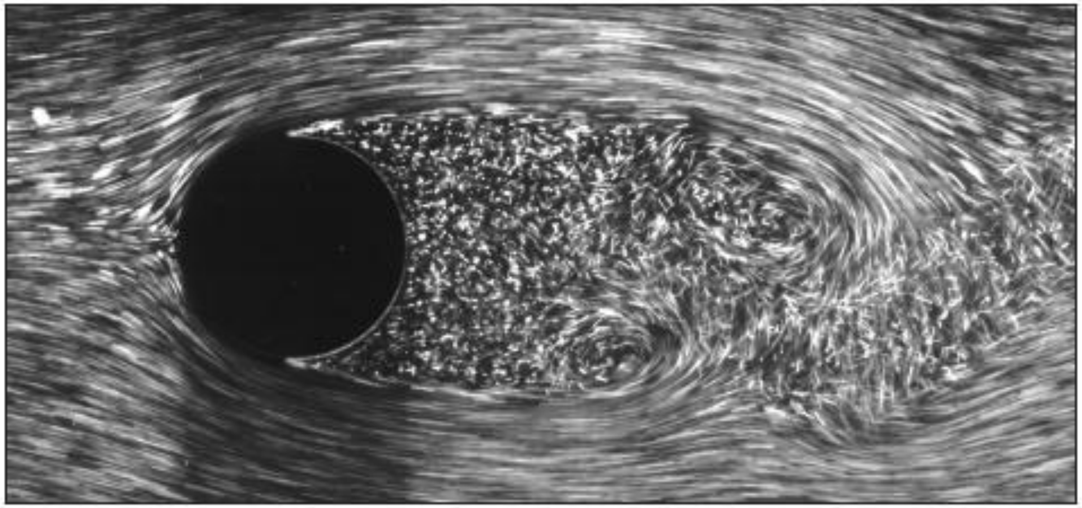
FLOW OVER CYLINDERS AND SPHERES

Flow over cylinders and spheres is frequently encountered in practice.

The tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes.

Many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $Re = VD/\nu$ where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $Re_{cr} \cong 2 \times 10^5$. That is, the boundary layer remains laminar for $Re \lesssim 2 \times 10^5$, is "transitional" for $2 \times 10^5 \lesssim Re \lesssim 2 \times 10^6$, and becomes fully turbulent for $Re \gtrsim 2 \times 10^6$.



At very low velocities, the fluid completely wraps around the cylinder. Flow in the wake region is characterized by periodic vortex formation and low pressures.

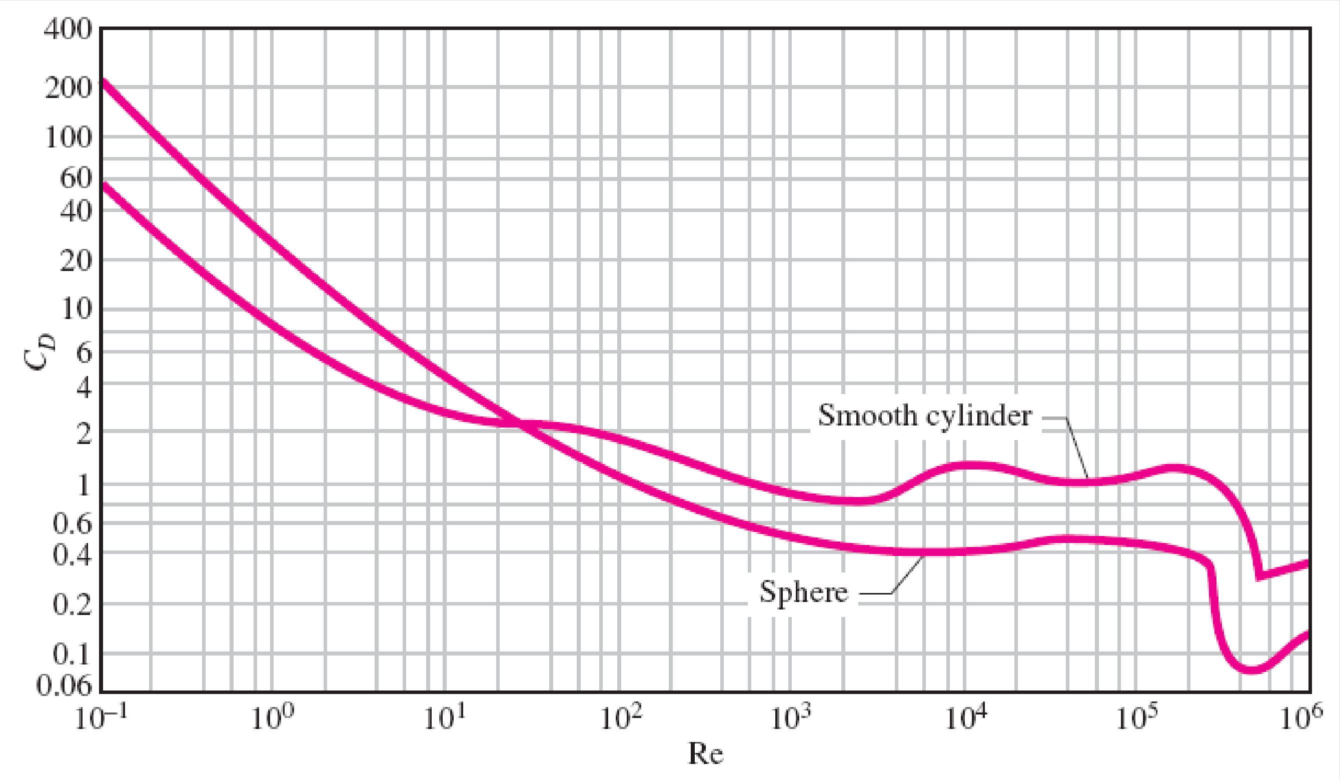
Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder at Re=2000.

For flow over cylinder or sphere, both the *friction drag* and the *pressure drag* can be significant.

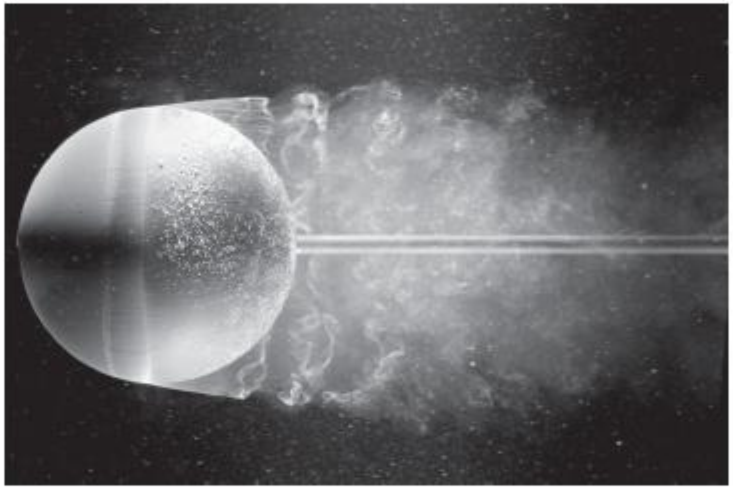
The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow.

The drag force is primarily due to friction drag at low Reynolds numbers (Re<10) and to pressure drag at high Reynolds numbers (Re>5000).

Both effects are significant at intermediate Reynolds numbers.



Average drag coefficient for cross-flow over a smooth circular cylinder and a smooth sphere.


(*a*)

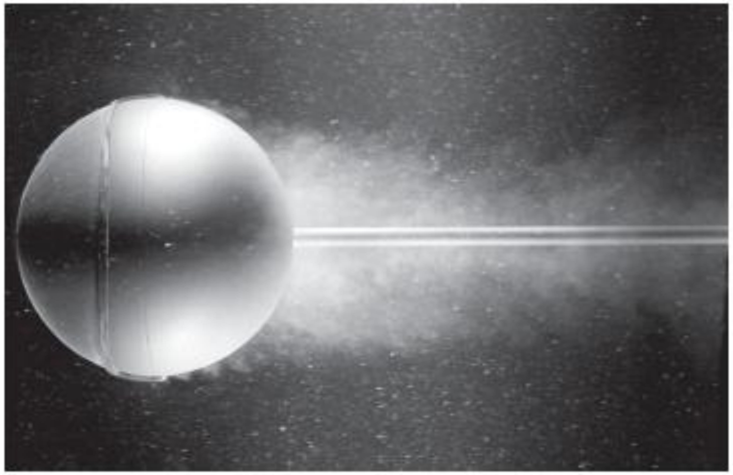

(*b*)

FIGURE 7–18
Flow visualization of flow over (*a*) a smooth sphere at Re = 15,000, and (*b*) a sphere at Re = 30,000 with a trip wire. The delay of boundary layer separation is clearly seen by comparing the two photographs.

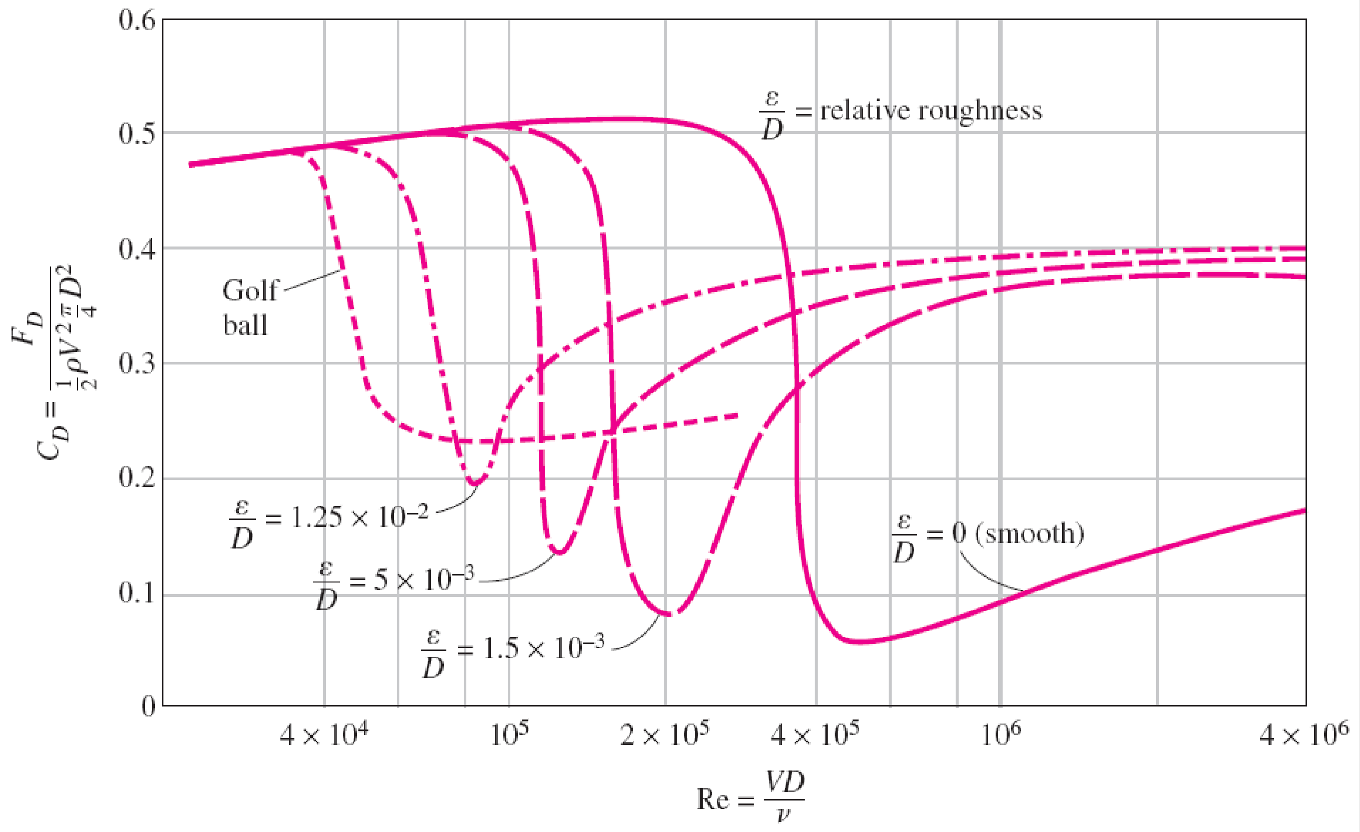
Flow separation occurs at about $\theta \cong 80°$ (measured from the front stagnation point of a cylinder) when the boundary layer is *laminar* and at about $\theta \cong 140°$ when it is *turbulent*

Effect of Surface Roughness

Surface roughness, in general, increases the drag coefficient in turbulent flow.

This is especially the case for streamlined bodies.

For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *increase* or *decrease* the drag coefficient depending on Reynolds number.



The effect of surface roughness on the drag coefficient of a sphere.

Re	C_D Smooth Surface	C_D Rough Surface, $\varepsilon/D = 0.0015$
2×10^5	0.5	0.1
10^6	0.1	0.4

FIGURE 7–20
Surface roughness may increase or decrease the drag coefficient of a spherical object, depending on the value of the Reynolds number.

EXAMPLE 7–4 Drag Force Acting on a Pipe in a River

A 2.2-cm-outer-diameter pipe is to cross a river at a 30-m-wide section while being completely immersed in water (Fig. 7–21). The average flow velocity of water is 4 m/s and the water temperature is 15°C. Determine the drag force exerted on the pipe by the river.

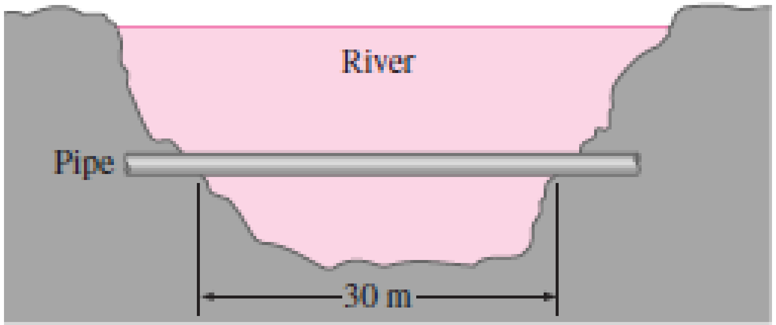


FIGURE 7–21
Schematic for Example 7–4.

SOLUTION A pipe is crossing a river. The drag force that acts on the pipe is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Figure 7–17 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and $\mu = 1.138 \times 10^{-3}$ kg/m · s (Table A-9).

Analysis Noting that D = 0.022 m, the Reynolds number for flow over the pipe is

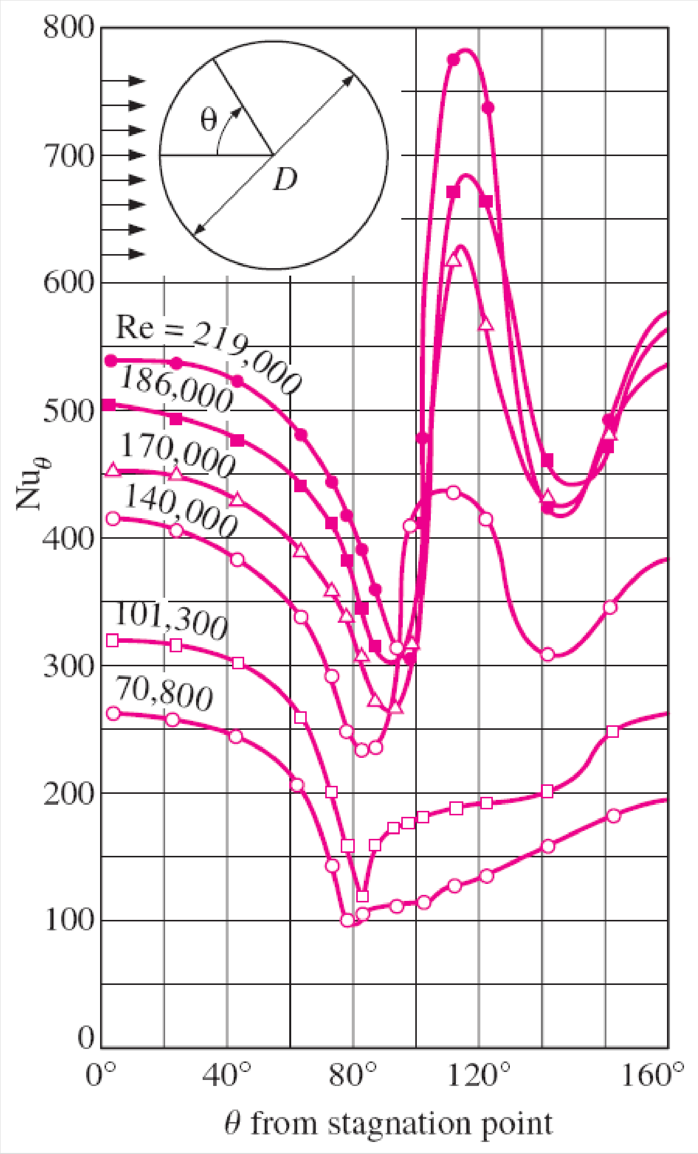
$$\mathrm{Re} = \frac{\mathcal{V}D}{\nu} = \frac{\rho\mathcal{V}D}{\mu} = \frac{(999.1\ \mathrm{kg/m^3})(4\ \mathrm{m/s})(0.022\ \mathrm{m})}{1.138 \times 10^{-3}\ \mathrm{kg/m \cdot s}} = 7.73 \times 10^4$$

The drag coefficient corresponding to this value is, from Figure 7-17, C_D = 1.0. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho\mathcal{V}^2}{2} = 1.0(30 \times 0.022\ \mathrm{m^2}) \frac{(999.1\ \mathrm{kg/m^3})(4\ \mathrm{m/s})^2}{2} \left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg \cdot m/s^2}}\right)$$
$$= 5275\ \mathrm{N}$$

Heat Transfer Coefficient

- Flows across cylinders and spheres, in general, involve *flow separation*, which is difficult to handle analytically.
- Flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.



Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross flow of air.

For flow over a *cylinder*

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$\mathrm{RePr} > 0.2$

The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$

For flow over a *sphere*

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$

The fluid properties are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s.

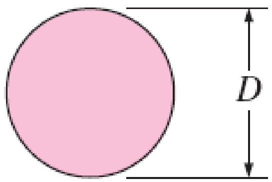
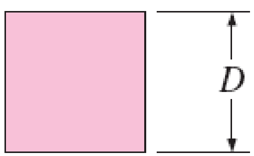
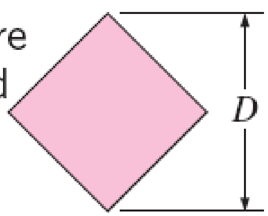
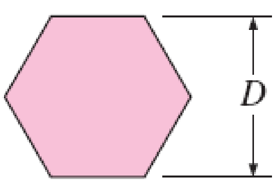
$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$n = \frac{1}{3}$

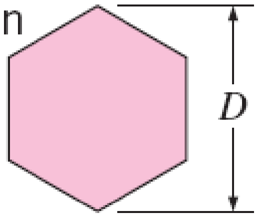
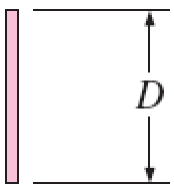
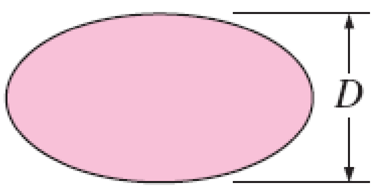
Constants *C* and *m* are given in the table.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. They are applicable to *smooth* surfaces.

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160 Re^{0.638} Pr^{1/3}$ $Nu = 0.0385 Re^{0.782} Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228 Re^{0.731} Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248 Re^{0.612} Pr^{1/3}$

EXAMPLE 7–5 Heat Loss from a Steam Pipe in Windy Air

A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds (Fig. 7–23). Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

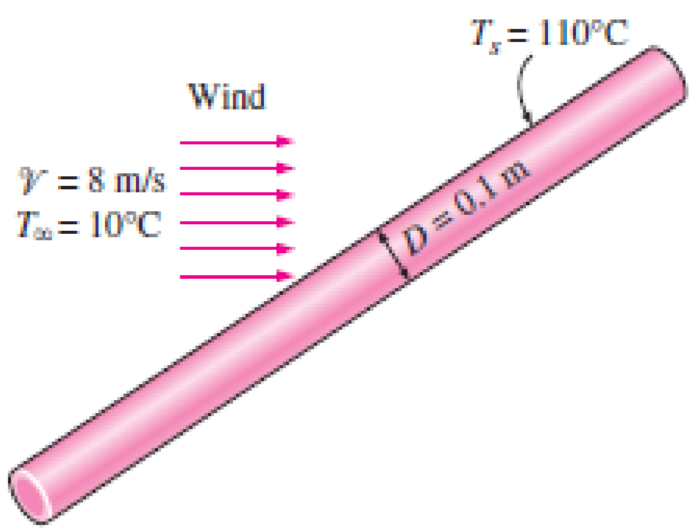


FIGURE 7–23
Schematic for Example 7–5.

SOLUTION A steam pipe is exposed to windy air. The rate of heat loss from the steam is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas.

Properties The properties of air at the average film temperature of $T_f = (T_s + T_\infty)/2 = (110 + 10)/2 = 60°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02808 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7202$$
$$\nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number is

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(8 \text{ m/s})(0.1 \text{ m})}{1.896 \times 10^{-5} \text{ m}^2/\text{s}} = 4.219 \times 10^4$$

The Nusselt number can be determined from

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \\ &= 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2}(0.7202)^{1/3}}{[1 + (0.4/0.7202)^{2/3}]^{1/4}}\left[1 + \left(\frac{4.219 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5} \\ &= 124\end{aligned}$$

and

$$h = \frac{k}{D}\text{Nu} = \frac{0.02808 \text{ W/m} \cdot °\text{C}}{0.1 \text{ m}}(124) = 34.8 \text{ W/m}^2 \cdot °\text{C}$$

Then the rate of heat transfer from the pipe per unit of its length becomes

$$A_s = pL = \pi DL = \pi(0.1 \text{ m})(1 \text{ m}) = 0.314 \text{ m}^2$$
$$\dot{Q} = hA_s(T_s - T_\infty) = (34.8 \text{ W/m}^2 \cdot \text{C})(0.314 \text{ m}^2)(110 - 10)°\text{C} = 1093 \text{ W}$$

The rate of heat loss from the entire pipe can be obtained by multiplying the value above by the length of the pipe in m.

EXAMPLE 7–6 Cooling of a Steel Ball by Forced Air

A 25-cm-diameter stainless steel ball ($\rho = 8055\ kg/m^3$, $C_p = 480\ J/kg \cdot °C$) is removed from the oven at a uniform temperature of 300°C (Fig. 7–24). The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.

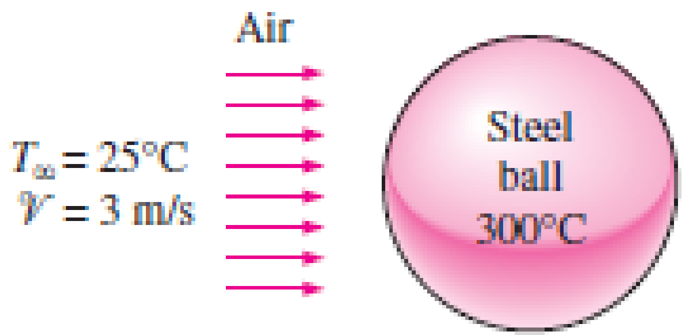


FIGURE 7–24
Schematic for Example 7–6.

SOLUTION A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\,250°C} = 2.76 \times 10^{-5}$ kg/m · s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A-15)

$$k = 0.02551 \text{ W/m} \cdot °\text{C} \qquad \nu = 1.562 \times 10^{-5} \text{ m}^2\text{/s}$$
$$\mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr} = 0.7296$$

Analysis The Reynolds number is determined from

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2\text{/s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 2 + [0.4 \,\text{Re}^{1/2} + 0.06 \,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ &= 2 + [0.4(4.802 \times 10^4)^{1/2} + 0.06(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \\ &\quad \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4} \\ &= 135\end{aligned}$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m}\cdot{}^\circ\text{C}}{0.25 \text{ m}}(135) = 13.8 \text{ W/m}^2\cdot{}^\circ\text{C}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25 \text{ m})^2 = 0.1963 \text{ m}^2$$

$$\dot{Q}_{ave} = hA_s(T_{s,\,ave} - T_\infty) = (13.8 \text{ W/m}^2\cdot{}^\circ\text{C})(0.1963 \text{ m}^2)(250 - 25)^\circ\text{C} = 610 \text{ W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho\tfrac{1}{6}\pi D^3 = (8055 \text{ kg/m}^3)\,\tfrac{1}{6}\pi(0.25 \text{ m})^3 = 65.9 \text{ kg}$$

$$Q_{total} = mC_p(T_2 - T_1) = (65.9 \text{ kg})(480 \text{ J/kg}\cdot{}^\circ\text{C})(300 - 200)^\circ\text{C} = 3{,}163{,}000 \text{ J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{ave}} = \frac{3{,}163{,}000 \text{ J}}{610 \text{ J/s}} = 5185 \text{ s} = 1 \text{ h } 26 \text{ min}$$

FLOW ACROSS TUBE BANKS

- Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment, e.g., heat exchangers.
- In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.
- Flow *through* the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes.
- For flow *over* the tubes the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them are altered.
- Typical arrangement: in-line or staggered
- The outer tube diameter *D* is the characteristic length.
- The arrangement of the tubes are characterized by the *transverse pitch* S_T, *longitudinal pitch* S_L, and the *diagonal pitch* S_D between tube centers.

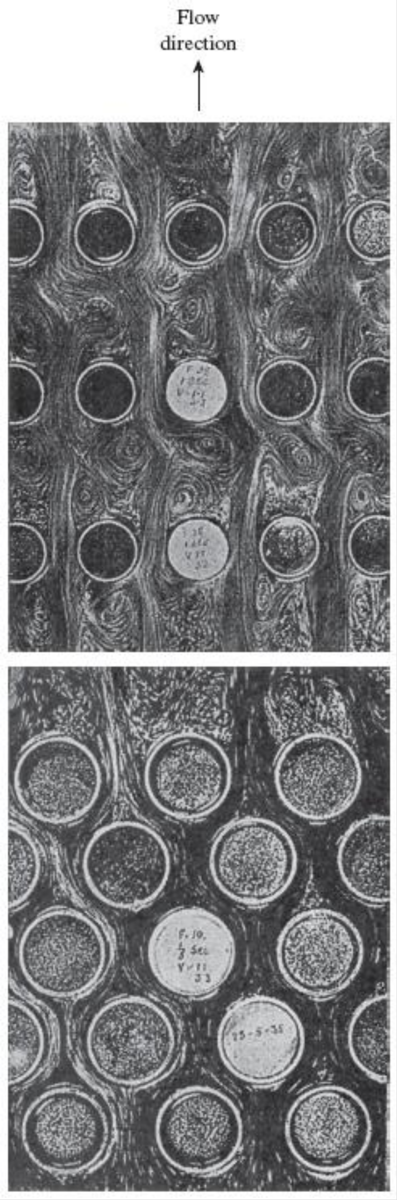


FIGURE 7–25
Flow patterns for staggered and in-line tube banks

$$S_D = \sqrt{S_L^2 + (S_T/2)^2}$$

diagonal pitch

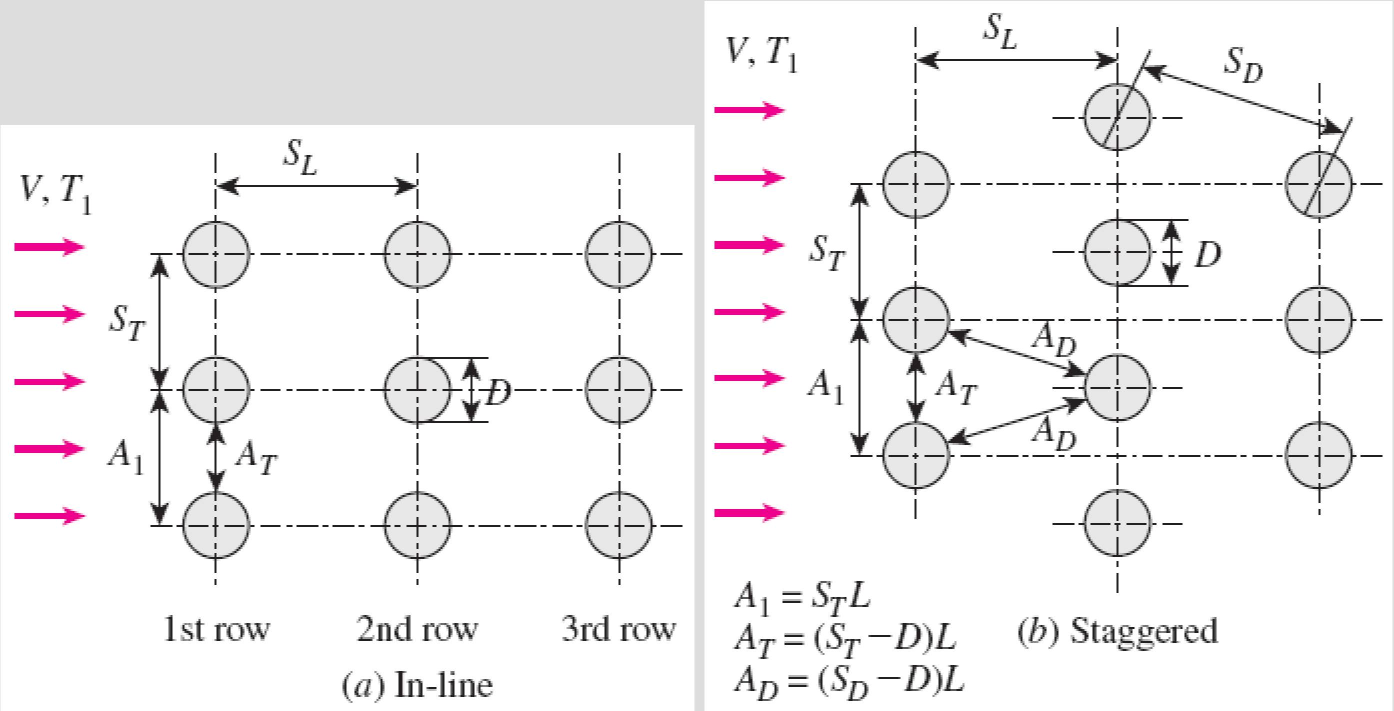
$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{V_{\max} D}{\nu}$$

$$\rho V A_1 = \rho V_{\max} A_T$$

$$V_{\max} = \frac{S_T}{S_T - D} V$$

Staggered and $S_D < (S_T + D)/2$: $$V_{\max} = \frac{S_T}{2(S_D - D)} V$$

$$\rho V A_1 = \rho V_{\max}(2A_D) \text{ or } V S_T = 2V_{\max}(S_D - D)$$



Arrangement of the tubes in in-line and staggered tube banks (A_1, A_T, and A_D are flow areas at indicated locations, and L is the length of the tubes).

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

Correlations in Table 7-2

$$T_m = \frac{T_i + T_e}{2}$$

All properties except Pr_s are to be evaluated at the arithmetic mean temperature.

The average Nusselt number relations in Table 7–2 are for tube banks with more than 16 rows. Those relations can also be used for tube banks with $N_L < 16$ provided that they are modified as

$$\mathrm{Nu}_{D,\,N_{L<16}} = F\,\mathrm{Nu}_D$$

N_L < 16

where F is a *correction factor* whose values are given in Table 7–3.

For $\mathrm{Re}_D > 1000$, the correction factor is independent of Reynolds number.

$$\Delta T_{\mathrm{lm}} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

Log mean temperature difference

Exit temperature

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m} c_p}\right)$$

$$A_s = N\pi DL$$

$$\dot{m} = \rho V(N_T S_T L)$$

$$\dot{Q} = hA_s\Delta T_{\mathrm{lm}} = \dot{m}c_p(T_e - T_i)$$

Heat transfer rate

$$Nu_D = \frac{hD}{k} = C\, Re_D^m\, Pr^n (Pr/Pr_s)^{0.25}$$

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for $N_L > 16$ and $0.7 < Pr < 500$ (from Zukauskas, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\, Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\, Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35 (S_T/S_L)^{0.2}\, Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031 (S_T/S_L)^{0.2}\, Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

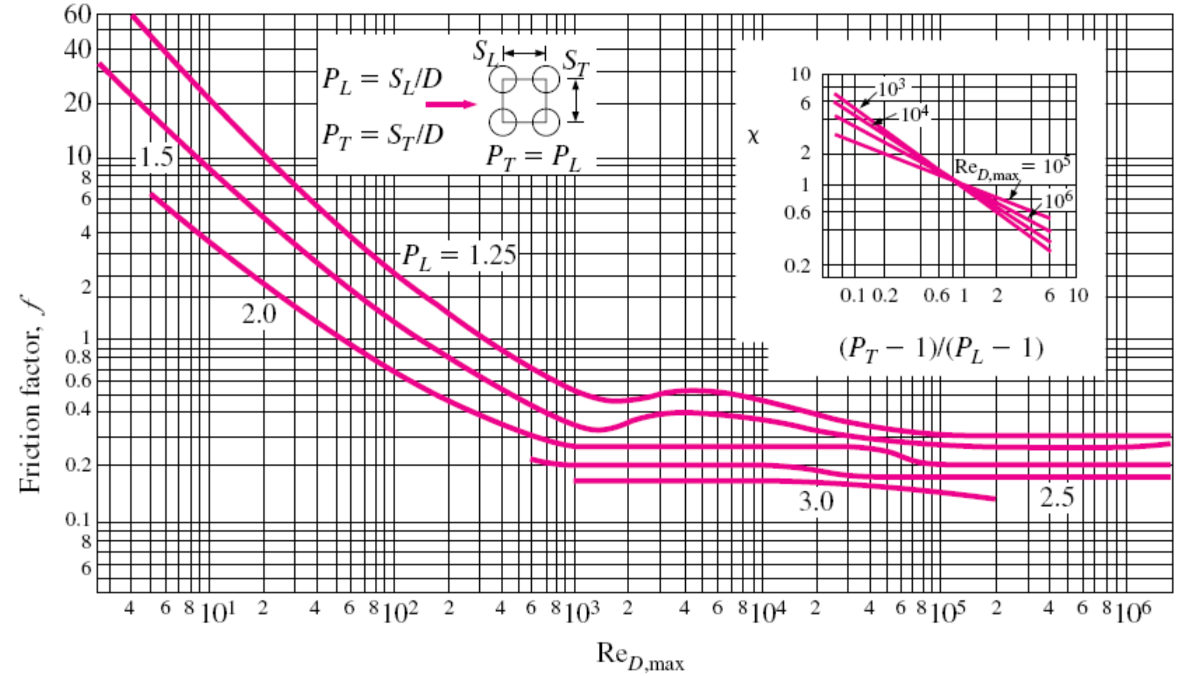
TABLE 7–3

Correction factor F to be used in $Nu_{D,\, N_L<16} = F\, Nu_D$ for $N_L > 16$ and $Re_D > 1000$ (from Zukauskas, 1987)

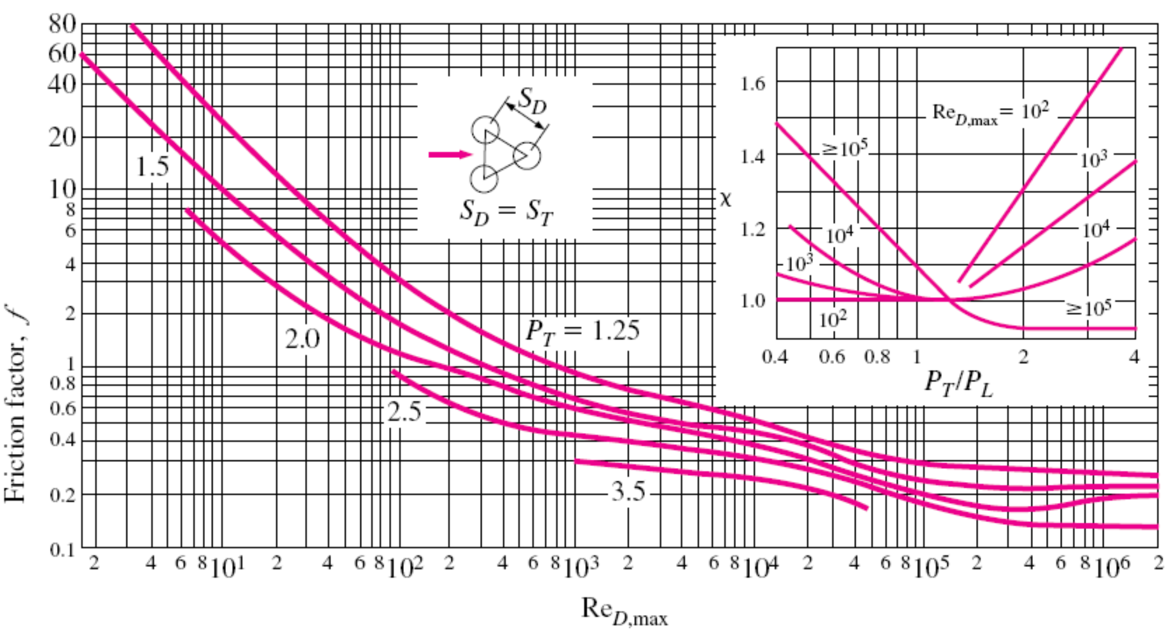
N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

$$Nu_{D,\, N_{L<16}} = F\, Nu_D$$

Pressure drop



(a) In-line arrangement



(b) Staggered arrangement

$$\Delta P = N_L f \chi \frac{\rho V_{\max}^2}{2}$$

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P = \frac{\dot{m} \Delta P}{\rho}$$

$$\dot{V} = V(N_T S_T L)$$

$$\dot{m} = \rho \dot{V} = \rho V(N_T S_T L)$$

- f is the friction factor and χ is the correction factor.
- The correction factor χ given is used to account for the effects of deviation from square arrangement (in-line) and from equilateral arrangement (staggered).

EXAMPLE 7–7 Preheating Air by Geothermal Water in a Tube Bank

In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120°C flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20°C and 1 atm with a mean velocity of 4.5 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 5$ cm. There are 6 rows in the flow direction with 10 tubes in each row, as shown in Figure 7–28. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.

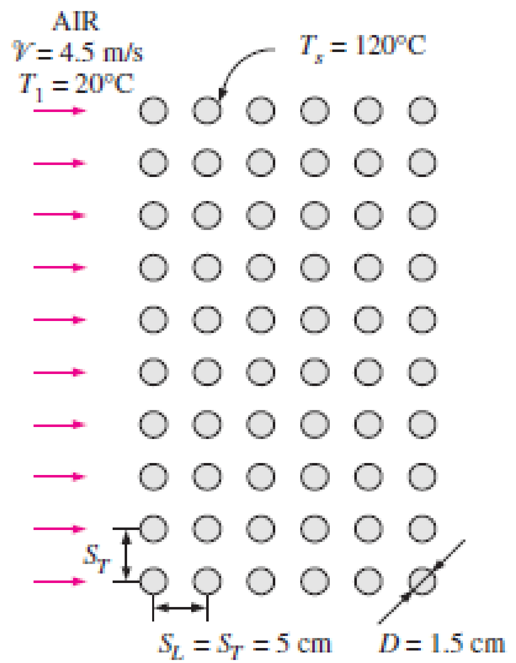


FIGURE 7–28
Schematic for Example 7–7.

SOLUTION Air is heated by geothermal water in a tube bank. The rate of heat transfer to air and the pressure drop of air are to be determined.

Assumptions 1 Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 60°C (will be checked later) and 1 atm are Table A–15):

$$k = 0.02808 \text{ W/m} \cdot \text{K}, \qquad \rho = 1.06 \text{ kg/m}^3$$
$$C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \qquad \text{Pr} = 0.7202$$
$$\mu = 2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr}_s = \text{Pr}_{@Ts} = 0.7073$$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204$ kg/m³

Analysis It is given that $D = 0.015$ m, $S_L = S_T = 0.05$ m, and $\mathcal{V} = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V} = \frac{0.05}{0.05 - 0.015}(4.5 \text{ m/s}) = 6.43 \text{ m/s}$$
$$\text{Re}_D = \frac{\rho \mathcal{V}_{\max} D}{\mu} = \frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 5091$$

The average Nusselt number is determined using the proper relation from Table 7–2 to be

$$\begin{aligned}\text{Nu}_D &= 0.27 \text{Re}_D^{0.63} \text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25} \\ &= 0.27(5091)^{0.63}(0.7202)^{0.36}(0.7202/0.7073)^{0.25} = 52.2\end{aligned}$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case, the number of rows is $N_L = 6$, and the corresponding correction factor from Table 7–3 is $F = 0.945$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D, N_L} = F\text{Nu}_D = (0.945)(52.2) = 49.3$$

$$h = \frac{\text{Nu}_{D, N_L} k}{D} = \frac{49.3(0.02808 \text{ W/m} \cdot {}^\circ\text{C})}{0.015 \text{ m}} = 92.2 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

The total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 60\pi(0.015 \text{ m})(1 \text{ m}) = 2.827 \text{ m}^2$$

$$\dot{m} = \dot{m}_1 = \rho_1 \mathcal{V}(N_T S_T L)$$
$$= (1.204 \text{ kg/m}^3)(4.5 \text{ m/s})(10)(0.05 \text{ m})(1 \text{ m}) = 2.709 \text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right)$$
$$= 120 - (120 - 20)\exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2 \cdot {}^\circ\text{C})}{(2.709 \text{ kg/s})(1007 \text{ J/kg} \cdot {}^\circ\text{C})}\right) = 29.11^\circ\text{C}$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{(120 - 29.11) - (120 - 20)}{\ln[(120 - 29.11)/(120 - 20)]} = 95.4^\circ\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (92.2 \text{ W/m}^2 \cdot {}^\circ\text{C})(2.827 \text{ m}^2)(95.4^\circ\text{C}) = 2.49 \times 10^4 \text{ W}$$

The rate of heat transfer can also be determined in a simpler way from

$$\dot{Q} = hA_s\Delta T_{\text{ln}} = \dot{m}C_p(T_e - T_i)$$
$$= (2.709\ \text{kg/s})(1007\ \text{J/kg}\cdot{}^\circ\text{C})(29.11 - 20)^\circ\text{C} = 2.49 \times 10^4\ \text{W}$$

For this square in-line tube bank, the friction coefficient corresponding to $\text{Re}_D = 5088$ and $S_L/D = 5/1.5 = 3.33$ is, from Fig. 7–27*a*, $f = 0.16$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f \chi \frac{\rho V_{\max}^2}{2}$$
$$= 6(0.16)(1)\frac{(1.06\ \text{kg/m}^3)(6.43\ \text{m/s})^3}{2}\left(\frac{1\text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 21\ \text{Pa}$$

Summary

- Drag and Heat Transfer in External Flow
  - ✓ Friction and pressure drag
  - ✓ Heat transfer
- Parallel Flow Over Flat Plates
  - ✓ Friction coefficient
  - ✓ Heat transfer coefficient
  - ✓ Flat plate with unheated starting length
  - ✓ Uniform Heat Flux
- Flow Across Cylinders and Spheres
  - ✓ Effect of surface roughness
  - ✓ Heat transfer coefficient
- Flow across Tube Banks
  - ✓ Pressure drop